For any quantity of interest in a system governed by nonlinear differential equations it is natural to seek the largest (or smallest) long-time average among solution trajectories. Upper bounds can be proved \emph{a priori} using auxiliary functions, the best choice of which is a convex optimization. We show that the problems of finding maximal trajectories and minimal auxiliary functions are strongly dual. Thus, auxiliary functions provide arbitrarily sharp upper bounds on maximal time averages. They also provide volumes in phase space where maximal trajectories must lie. For polynomial equations, auxiliary functions can be constructed by semidefinite programming which we illustrate using the Lorenz and Kuramoto-Sivashinsky equations. This is joint work with Ian Tobasco and David Goluskin, part of which appears in \textit{Physics Letters A} \textbf{382}, 382–386 (2018).